An Introduction to the Volatility Smile

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Aim of the Course

This is a course about several themes:
1. Understanding volatility as a quality, a quantity, and an asset
2. Understanding the practical use of the Black-Scholes-Merton model. There’s more to it than just knowing the equation and its solution.
3. Understanding the successes and limitations of the Black-Scholes model.
4. Coming to grips with the volatility smile.
5. The extensions of the Black-Scholes model to accommodate/explain the volatility smile.
6. Understanding the consequences of these extensions. It’s easy to make up new and richer models but we want to understand whether they are realistic, whether they are advantageous, and what they lead to.

This course uses mathematics, but it isn’t a course about mathematics, differential equations or stochastic calculus. The aim is to use mathematics to understand the volatility smile. I want to not merely study the solutions and methods of solution of a variety of options models, but also develop intuition about what they suppose and how to use them. No assumptions behind financial models are genuinely true, and no financial models are really correct, so it’s very important to understand what you’re doing and why you’re doing it.

I’ll try to approach the models with a mixture of theory and pragmatism. I don’t like simply writing down formulas without proofs, though I’ll do that occasionally. Knowing the formulas like a table of integrals isn’t enough when you’re working in the field, because many of the derivative products you have to deal with, and their markets, violate the assumptions behind the simple formulas. One needs to develop intuition about the models, so that you can know when your calculations are giving you the right answer or when you’ve made a mathematical or computational or programing mistake.

So, I’ll put a lot of effort into deriving simple or approximate proofs of the key model formulas and ideas. Often these proofs and formulas may not be the best way to implement a model for rapid and accurate computational use, but they can be good for understanding ideas. My aim is to develop these models logically, to get a feel for the phenomena to be explained, and to estimate the effects of the models.
Course Outline

• The Principles of Financial Modeling
  Viewpoint: relative rather than absolute valuation
  The foundations of financial theory and valuation
  The theory of dynamic hedging.

• Option Valuation
  The theory of dynamic replication
  The Black-Scholes equation
  Variance swaps
  P&L (profit and loss) of options trading

• Introduction to the Implied Volatility Smile
  The smile in various markets
  The difficulties the smile presents for trading desks and for theorists
  Reasons for a smile
  No-riskless-arbitrage bounds on the size of the smile

• Implied Distributions Extracted from the Smile
  Breeden Litzenberger formula
  Static replication of path-independent exotic options with vanilla options

• Static Replication
  Approximate static replication of path-dependent exotic options with vanilla options

• Extending Black-Scholes beyond constant-volatility lognormal stock price evolution
  Binomial trees
  Time-dependent deterministic rates
  Time-dependent deterministic volatility
  Alternative stochastic processes that could account for the smile

• Local Volatility Models/ Implied Trees
  Derman-Kani binomial local volatility trees
  Dupire equation
  Calibration of implied binomial trees
  How to build an implied tree from options prices.

• The Consequences of Local Volatility Models
  The local volatility surface
  The relationship between local and implied volatility
  Estimating the deltas of vanilla options in the presence of the smile
  Estimating the values of exotic options
• **Stochastic Volatility Models**
  The effect of changes in volatility in the Black-Scholes formula
  The Vanna-Volga way of looking at things
  The SABR model
  The PDE for option value under stochastic volatility
  The mixing formula for option value under stochastic volatility
  Estimating the smile in stochastic volatility models
  The relationship between local and stochastic volatility

• **Jump-Diffusion Models**
  Are they reasonable, and if so, when?
  The Merton jump-diffusion model and its solution
  Estimating the smile in jump-diffusion models